Letters

Comments on "A New Finite-Difference Time-Domain Formulation and its Equivalence with the TLM Symmetrical Condensed Node"

M. Celuch-Marcysiak and W. K. Gwarek

In [1] the authors present a new formulation of the finite-difference time-domain (FD-TD) method for Maxwell's equations. They compare this formulation (which we shall call "New FD-TD") with other existing FD-TD and TLM algorithms. They come to conclusions which can be summarized in the following statements:

1. For the 2-D case the classical FD-TD formulation is not equivalent to the classical TLM shunt-node or series-node formulation. New FD-TD is equivalent to the classical TLM shunt-node algorithm. Shunt-node TLM or its equivalent—new FD-TD provide more accurate description of the electromagnetic field than classical FD-TD. Consequently, they produce the analyzed circuit parameters with better accuracy (for example eigenvalues presented in Table I of [1]).
2. Still considering the 2-D case, the TLM and new FD-TD calculations converge more rapidly to the final solution when the number of iterations increases, in comparison with classical FD-TD (see Fig. 6 in [1]).
3. These advantages of TLM and new FD-TD over classical FD-TD result from the fact that they automatically provide "energy conservation within cubic cells, thus generating a stable nondissipative solution" while "energy conservation must be explicitly enforced in a FD-TD scheme to ensure it."
4. The other reason for better accuracy of TLM and new FD-TD in comparison with classical FD-TD is that "both electric and magnetic field components are defined at the cell centers and at mid-points between adjacent cells."
5. The new FD-TD algorithm extended to three dimensions becomes fully equivalent to Symmetrical Condensed Node TLM. When compared with the classical three—dimensional FD-TD formulation, new FD-TD and SCN-TLM retain the advantage of automatically conserving energy and of better field resolution (like in 2-D), and on these grounds better accuracy is also expected.

Neither of the statements 1–4 is entirely correct and statement 5 contains misleading interpretations of the differences between the SCN TLM node and the classical Yee's mesh. We will discuss them in detail.

I. Comments on Statement 1

The classical 2-D FD-TD and the shunt node TLM are formally equivalent in application to the analysis of circuits of arbitrary complexity. It means that no extra accuracy of calculation of circuit parameters can result from substituting the FD-TD wave simulation by the TLM wave simulation. This is the essence of the Equivalence Theorem presented in [4] and proved in [5]. To avoid confusion in further discussion, let us here clearly specify the conditions for this equivalence. A fundamental TLM mesh is described by four quantities (e.g., four incident pulses) changing after each iteration. Let us call them W_k with $k = 1, 2, 3, 4$. In a similar way a fundamental FD-TD mesh is described by three quantities (one nodal voltage and two branch currents) which we will call U_1 with $l = 1, 2, 3$. We characterize the analyzed circuit by a transmittance operator T relating an input quantity $V_{\text{in}}(t)$ and an output quantity $V_{\text{out}}(t)$.

$$V_{\text{out}}(t) = T[V_{\text{in}}(t)] \tag{1}$$

The operator T is obtained by FD-TD calculations (we will call it T_f) or by TLM calculations (we will call it T_t). The Equivalence Theorem states that if V_{in} and V_{out} are chosen in such a way that they can be expressed as linear combination of the quantities W_k and also as linear combinations of U_1, then the operators T_f and T_t are nearly identical, that is they differ only by the computer round off errors. For both methods these errors are very small and usually several orders of magnitude below the acceptable errors of analysis. Thus, contrary to the suggestions expressed in [1], the differences between FD-TD and TLM results presented in Table I of the paper cannot be attributed to the differences in the model of propagation. We would rather attribute them to the differences in excitation conditions, as pointed out in our further comments.

For completeness of our discussion we must consider that TLM contains some information not explicitly available in classical FD-TD. This additional set of information consists of nodal currents and branch voltages. Therefore while all the quantities U_l can be expressed as linear combinations of W_k, not all the W_k can be expressed as linear combinations of U_l. Thus a question may arise about expressing (1) when V_{out} and V_{in} are quantities available in TLM but not explicitly available in FD-TD. It can be shown [4], [5] that we can obtain these quantities in the classical FD-TD by adding few simple operations. In particular, the repeated space averaging of standard FD-TD nodal voltages produces branch voltages fully equivalent to TLM branch voltages. In fact, governing (14)–(17) in paper [1] can also be used as generators of auxiliary information. The relative advantage of the repeated space averaging of [4], [5] is that it can be easily applied locally, for example only in a single mesh at input and a single mesh at output, just where we actually need to decompose V_{in} and V_{out} into quantities used by the simulation algorithm. There is no need to add any operations in all other meshes.

In view of the Equivalence Theorem and its consequences mentioned above, it is not possible for the new FD-TD algorithm to be equivalent to classical TLM and not equivalent to classical FD-TD. We find it straightforward to show that—contrary to the conclusions of [1]—all three algorithms are equivalent, although indeed each uses a different number of variables to describe the electromagnetic field over a fundamental mesh and, consequently, a different number of equations to update the field after each iteration.

II. Comments on Statement 2

To show that the TLM algorithm converges more quickly than the FD-TD algorithm the authors of [1] present an example of eigenvalue calculations (Fig. 6) in which after 200–300 iterations the error in FD-TD is of the order of 5% of the final value while in TLM—of the

Manuscript received April 16, 1992.

The authors are with the Institute of Radio-Electronics, Warsaw University of Technology, 00–665, Warszawa, Nowowiejska, 15/19, Poland.

IEEE Log Number 9204039.

order of 1%. Now, an important aspect of the Equivalence Theorem [4], [5] is that it does not only state the equivalence of T_f and T_t operators which are obtained as final results of the FD-TD and TLM algorithms, respectively but also the equivalence of corresponding field components at each individual mesh and at each individual iteration. Thus at any location in the circuit and at any time instant the values of a pair of corresponding FD-TD and TLM functions (e.g., nodal voltages) differ only by the computer round off errors. this means in particular that FD-TD and TLM have to converge along the same path. In other words, the differences of convergence properties such as those exhibited in Fig. 6 of [1] can by no means be attributed to the mere change from the TLM to FD-TD simulation algorithm.

Obviously, the validity of our above conclusion requires the use of equivalent models of circuit boundaries and of excitation. It can be shown [4], [5] that any models of space discontinuities or of excitation functions existing in either the TLM or FD-TD notation—can be equivalently transformed into the other notation. Therefore the requirement of equivalent excitation and boundary conditions can always be satisfied, leading to the meaningful comparison of the actual numerical properties of TLM and FD-TD, which then clearly prove equivalent. This we have verified for a great variety of circuits of most sophisticated shapes, filling and excitation. On the other hand, we have found that the speed of convergence strongly depends on several other factors, such as:

- A choice of the function $f(w)$ to be investigated in the search of eigenvalues; some functions produce maxima and some of them minima for eigenvalues and the extrema are not equally sharp;
- A choice of the positioning of input and output meshes to define $f(w)$;
- A choice of the source of excitation [6].

Not knowing the exact conditions under which the calculations of Fig. 6 have been performed we cannot repeat them exactly. However, using the grid presented in Fig. 5(b) of [1] we have performed eigenvalue calculations using a classical Yee's FD-TD mesh and we have obtained the transient response error falling below 0.3% after 150 iterations. If this is still not convincing, we would appreciate sending us the exact description of the conditions under which the calculations of Fig. 6 have been performed and we will show how to make classical FD-TD perform like TLM, or vice versa.

III. Comments on Statement 3

An exact mathematical way of verifying whether a numerical scheme conserves energy or whether it is dissipative—consists in analyzing its dispersion relations [10]. If for all real values of a propagation constant the numerical scheme gives purely real values of frequency, then the conservation of energy is guaranteed. Dispersion analysis of the FD-TD algorithm is a straightforward task and has been performed by several authors, e.g., [7]. It is easily seen that classical FD-TD naturally produces stable and nondissipative solutions. This in fact is a direct consequence of using central finite differences in both space and time [10]. Therefore contrary to Statement 3., explicit enforcement of energy conservation into FD-TD is unnecessary and redundant in terms of overall accuracy of modeling the propagation space. This justifies in particular why all three algorithms: classical FD-TD, TLM and New FD-TD (as presented in [1] for the 2-D case) are formally equivalent, although only the latter two explicitly incorporate the energy criterion. Clearly, explicit enforcement into a numerical scheme of a condition which is inherent in this scheme anyway—cannot be expected to improve the accuracy of calculations.

IV. Comments on Statement 4

Formal equivalence of classical FD-TD, TLM and new FD-TD means that contrary to the suggestions made in [1], the calculation of more field components by the latter two algorithms and the positioning of electric and magnetic field components at common locations do not improve the overall accuracy of the analysis. Essentially, the accuracy is determined by the underlying scheme of numerical integration in space and time. It is true that the positioning of field components to some extend reveals the scheme of integration—but by no means determines it. This will be exemplified in our further comments.

V. Comments on Statement 5

In 3D, the authors of [1] suggest that the differences between the condensed node models and the classical Yee's mesh result simply from the different number and positioning of the calculated field components and from the fact that the energy conservation criterion is included explicitly in the condensed node algorithms. Like in 2D, we claim that such an interpretation is invalid.

Paradoxically, the New FD-TD of [1] in itself provides a very good example to support our claim. The authors of [1] are right in pointing out that their new FD-TD in 3D is equivalent to the Symmetrical Condensed Node (SCN) formulation of TLM. But because of this equivalence their 3-D algorithm applied to the case of 2-D propagation must be different from the previously discussed new 2-D FD-TD (which is equivalent to the conventional shunt node TLM). We shall show this difference in a formal way.

The New 2-D FD-TD introduced in Section II-A of [1] (which we will from now on call "New 2-D FD-TD Type A"), is an equivalent of a classical shunt node TLM formulation, which in vacuum can be described by the following scattering matrix [9]:

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{bmatrix}^{t_0+\Delta t} = 1/2 \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \\ V_4^+ \end{bmatrix}^{t_o} \tag{2}$$

Now we consider the New 3-D FD-TD introduced in Section II-B of [1] and reduce it to the two dimensional wave propagation. What we obtain is a different 2-D version which we will call New 2-D FD-TD Type B. The version B is equivalent to a 2-D version of the Symmetrical Condensed TLM node which in vacuum can be characterized by a pair of scattering matrices:

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{bmatrix}^{t_0+\Delta t} = 1/2 \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \\ V_4^+ \end{bmatrix}^{t_0} + 1/2 \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \\ V_4^+ \end{bmatrix}^{t_o-\Delta t} \tag{3}$$

The most important consequence of the additional time delay produced by (3) with respect to (2) is the well known effect of zero dispersion in the axial direction of the SCN modeling.

Let us now get a closer look at the New FD-TD algorithms of Type A and Type B. They use the same number of field components situated in the same way. They also use the same criterion of energy conservation (see (10) . . . (12) and (29) . . . (32)). And yet the algorithm of Type A produces results identical to the classical Yee's FD-TD while the algorithm of Type B clearly has different properties, including zero dispersion for axial propagation. The authors of [1]

have neglected to pay attention to this fundamental discrepancy between their two algorithms.

Let us briefly point out that the only reason for this discrepancy consists in the numerical value of nodal inductance $L(L = mi$ in Type A and $2L = mi$ in Type B) which effectually determines the scheme of numerical integration. In [1] the two different values of L are used without any discussion or interpretation.

VI. Conclusions

On the base of the discussion presented above let us now formulate some general conclusions:

1. The FD-TD and TLM algorithms which have been reported up to now can be divided into two groups:
   Group A of expanded modeling which includes:
   a. Classical Yee's FD-TD,
   b. Classical TLM shunt or series node in 2-D;
   c. Expanded Node 3-D TLM;
   d. Spatial Network Method;
   e. New 2-D FD-TD of [1] which we call "type A";
   f. Extended FD-TD of [5].

   Group B of condensed modeling which includes:
   a. Symmetrical Condensed TLM [2];
   b. New 3-D FD-TD of [1];
   c. New 2-D FD-TD of [1] called "Type B" in this paper.

   Within each of the groups all the algorithms provide the same accuracy for analyzing circuits of any complexity, with any boundary conditions and any excitation. However, the algorithm significantly differ in terms of the computer effort required to obtain the solution.
2. The properties of the algorithms of Group A are very well investigated. This cannot be said about the properties of the algorithms of Group B. While they generally give a smaller frequency error [3], they are also prone to some types of parasitic solutions [8]. Furthermore, the Group B algorithms experience a more severe efficiency bound than the Group A algorithms, that is, for any particular problem the most effective algorithm from Group B requires more computer resources than the most effective algorithm from Group A.

References

[1] Z. Chen, M. M. Ney, and W. J. R. Hoefer, "A new finite-difference time-domain formulation and its equivalence with the TLM symmetrical condensed node." *IEEE Trans Microwave Theory Tech.*, vol. 39, no. 12, pp. 2160–2169. Dec. 1991.
[2] P. B. Johns, "A symmetrical condensed node for the TLM method," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 370–377. Apr. 1987.
[3] R. Allen, A. Malık, and P. B. Johns, "Numerical results for the symmetrical condensed TLM Node," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 378–382. Apr. 1987.
[4] M. Celuch-Marcysiak, and W. K. Gwarek, "Formal equivalence and efficiency comparison of the FD-TD, TLM and SN methods in application to microwave CAD programs," *Proc. XXI European Microwave Conf.*, Stuttgart, 1991, pp. 199–204.
[5] M. Celuch-Marcysiak and W. K. Gwarek, "Comparative analysis of the FD-TD, TLM and SN methods applied to boundary problems for Maxwell's equations," *submitted to Int. J Numerical Modeling.*
[6] W. K. Gwarek, "On the proper choice of excitation in the analysis of electromagnetic eigenvalue problems by finite-difference time-domain method," in *Proc. European Conf. on Numerical Methods in Electromagnetism, "Numelec,"* Grenoble, Mar. 1992.
[7] N. R. S. Simons, E. Bridges, "Equivalence of propagation characteristics for the TLM and FD TD methods in two dimensions," *IEEE Trans Microwave Theory Tech.*, vol. 39, no. 2, pp. 354–357, Feb. 1991.
[8] P. Ayoagi and R. Mittra, "On a correction to the voltage/field relationship used in the condensed node TLM method," in *IEEE Antennas Propagat. Symp. Dig.*, Dallas, 1990, pp. 10–13.
[9] W. J. R. Hoefer, "The transmission line matrix (TLM) method, in *Numerical Techniques for Microwave and Millimeter-Wave Passive Structures*, T. Itoh, Ed., ch. 8, New York: Wiley, 1989.
[10] D. Potter, *Computational Physics*, ch. 6. New York: Wiley, 1973.

Reply to Comments on "A New Finite-Difference Time-Domain Formulation and its Equivalence with the TLM Symmetrical Condensed Node"

Zhizhang Chen, Michel M. Ney and Wolfgang J. R. Hoefer

The above comments on our paper give us a welcome opportunity to clarify some of the points raised by Celuch-Marcysiak and Gwarek.

These comments begin with five statements which Celuch-Marcysiak and Gwarek refer to as *our conclusions*. However, we would like to remind the reader that these are clearly *conclusions attributed to us* by Celuch-Marcysiak and Gwarek. Our own conclusions are, of course, those in our paper [1]. Nevertheless, the five statements by Celuch-Marcysiak and Gwarek are good starting points for a discussion. We will therefore deal with them point by point. In their comments, Celuch-Marcysiak and Gwarek refer repeatedly to an equivalence theorem which is yet to be published [2]. We are therefore not in a position to comment specifically on statements made by the authors based on that paper. Nevertheless, we will address the conclusions drawn by the authors regarding our findings.

I. Comment on Statement 1

The FD-TD scheme described in [3] by Celuch-Marcysiak and Gwarek uses an average procedure on the branch voltages (see Eq. (9) in [3]). From their equivalence theorem [2] they conclude that the operators are "nearly" identical. Although the issue of equivalence between Yee's scheme and 2-D TLM was not addressed explicitly in our paper [1], we agree that, in free space, the 2-D TLM node algorithm can indeed be made equivalent to Yee's algorithm (at the limit of the stability criterion) by considering the branch current (TM-case) half-way between the nodes and taking the branch voltages at the same location as redundant quantities. In this case, both algorithms are equivalent. Unfortunately, this equivalence breaks down when boundaries, interfaces and sources (all necessary for practical modeling) are introduced. As a result, some additional operations on the classical Yee's scheme are needed

Manuscript received April 30, 1992.
Z. Chen and M. M. Ney are with the Laboratory for Electromagnetics and Microwaves, University of Ottawa, Ottawa, ON, Canada, K1N 6N5.
W.J.R. Hoefer is with the Department of Electrical and Computer Engineering, University of Victoria, Victoria, BC, Canada, V8W 3P6.
IEEE Log Number 9204040.

to ensure mathematical equivalence with the 2-D TLM. This point has already been explained by Johns in his reply [4]. In their paper [3], Celuch-Marcysiak and Gwarek also mentioned on p. 202 that formal equivalence requires the repeated space (and, in fact, time) averaging added to the classical Yee's scheme. The objective in [1] was to establish new FD-TD formulations which are exactly equivalent to the 2-D TLM shunt node and to the 3-D TLM condensed node, respectively, for any situation and in all respects.

II. Comment on Statement 2

We acknowledge that different excitations will produce different time-domain output and, therefore, different spectra. However, we think that the statement that if the transient response is falling below 0.3% (of the maximum value, we suppose), the resonant frequency is accurate within the same percentage after 150 iterations is not conclusive. The truncation error that shifts the maxima corresponding to resonant frequencies in the frequency domain must be taken into account. Therefore, a transient response close to zero is not a sufficient condition to ensure that the resonant frequency is obtained with good accuracy. We would be interested in the Fourier transform spectrum obtained by Celuch-Marcysiak and Gwarek to ascertain their conclusion. In addition, the structure cannot, in general, be excited in the same way when using TLM or classical Yee's scheme. For instance, in the TLM model the node at the center of the structure of Fig. 6 in [1] is excited by an impulsive branch voltage and zero currents while this cannot be done in the classical Yee's scheme. This may cause differences in the time-response but cannot entirely explain the difference in the convergence behavior. Our hypothesis, as mentioned in [1], is that explicit enforcement of the energy conservation in the new FD-TD makes the new algorithm numerically robust and attenuates the numerical noise during simulations, especially in the case of a relatively coarse mesh as used in our example. However, the point raised by Celuch-Marcysiak and Gwarek is noteworthy, and more study is needed to clearly identify the exact reasons for the difference in convergence behavior of the schemes.

III. Comment on Statement 3

There is a fundamental difference between the classical Yee's scheme and the 2-D TLM algorithm. The former approximates Maxwell's equations in discrete form by using difference operators which, inherently, enhance numerical noise. As a result, if Maxwell's equations implicitly satisfy the energy conservation principle, a discrete form such as Yee's scheme will only approximately satisfy it. On the other hand, TLM and the new FD-TD scheme proposed in [1] compute the new field values using a scattering process which explicitly enforces energy conservation. The behavior of the TLM simulation suggests that explicit enforcement of energy conservation leads to a faster convergence. This is consistent with the well-known use of redundancy in communications to reduce the error rate due to noise.

IV. Comment on Statement 4

We still maintain that the presence of more field components allows a more accurate description of the boundaries and excitation. We refer again to the reply by Johns [4] along these lines.

V. Comment on Statement 5

It is not clear to us what claim is brought forward by Celuch-Marcysiak and Gwarek. While we suggested in our paper [1] that the improved convergence behavior observed in the 2-D TLM and the new 2-D FD-TD scheme may be due to the explicit enforcement of energy conservation and conservation of all tangential field components, we never claimed that this was the reason for the zero dispersion along the main axes in 3-D condensed node algorithms. Nor did we neglect to pay attention to this fundamental difference between our two algorithms. It is simply a well-known fact which our research group has already analyzed in great detail [5], [6]. The explanation given by Celuch-Marcysiak and Gwarek in their comments thus stands entirely on its own merits. We have never made any statement to the contrary.

We acknowledge the new type of 2-D TLM node (referred to as type B) proposed by the authors, which is, as mentioned above, a reduction of the 3-D TLM condensed node and whose origin, surprisingly, is attributed to us by the authors. We acknowledge the expected nondispersive character in the axial direction of the node governed by equation (3) of their comments, but have some reservations regarding the claim that this algorithm has the same explicit energy conservation criterion as the 2-D TLM or FD-TD of [1] (referred to as type A). The authors failed to notice that their new algorithm (3) requires two time steps for updating the field values. On the other hand, the 2-D algorithms presented in [1] state explicitly that energy emerging from the node for new field updates is equal to the energy that entered it *at the previous time step only*. The new 2-D node which the authors are proposing is certainly worth investigating, and it would be interesting to compare it with the 2-D nodes presented in [1] or with Yee's scheme. For instance, even though no dispersion occurs in the axial direction, the node may still exhibit some dispersion along other directions.

VI. Conclusions

1. We do not agree with the classification of the various time-domain schemes presented by Celuch-Marcysiak and Gwarek in their conclusion. For instance, the term "expanded node" designates, according to the generally accepted definition, a node in which the field components are defined at different locations within the node. The 2-D TLM shunt node and the 2-D FD-TD node type A do not belong to that category. All field components are computed exactly at the same location, which is not the case in the classical Yee's scheme.
2. Celuch-Marcysiak and Gwarek point out that the algorithms of Group A have different computer expenditures. How, then, can they be formally equivalent?
3. The properties of the node proposed by Celuch-Marcysiak and Gwarek (called type B) remains to be investigated. The new 3-D FD-TD scheme described in [1] has exactly the same properties as the 3-D TLM condensed node whose properties, to the contrary of the authors claim, have been thoroughly studied (see for instance [5]–[7]). We agree that spurious mode solutions arise in the group B nodes, but they are also present in the algorithms of group A. These are inherent to the space and time discretization process and have noticeable effects when active circuits and absorbing boundaries are implemented. Again, due to the mathematical difference of the various algorithms, spurious modes have different properties and are excited in different ways. While some algorithms of group B require more computer storage, their faster convergence makes their CPU time requirement comparable. Finally, explicit enforcement of energy conservation ensures long term numerical stability and convergence.

References

[1] Z. Chen, M. M. Ney and W. J. R. Hoefer, "A new finite-difference time-domain formulation and its equivalence with the TLM symmetrical condensed node," *IEEE Trans. Microwave Theory Tech.*, vol. 39, no. 12, pp. 2160–2169, Dec. 1991.

[2] M. Celuch-Marcysiak and W. K. Gwarek, "Comparative analysis of the FD-TD, TLM and SN methods applied to boundary problems for Maxwell's equations," submitted for publication to *Int. J. Numerical Modeling*.
[3] ——, "Formal equivalence and efficiency comparison of the FD-TD, TLM, and SN Methods in application to microwave CAD program," in *Proc. 21th European Microwave Conf.*, Stuttgart, 1991, pp. 199–204.
[4] W. K. Gwarek, Comments on "On the relationship between TLM and finite-difference methods for Maxwell's equations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, no. 9, Sept. 1987, pp. 872–873.
[5] J. S. Nielsen, W. J. R. Hoefer, "A complete dispersion analysis of the condensed node TLM mesh," in *4th Biennial IEEE Conf. Electromagnetic Field Computation Dig.*, Toronto, ON, Oct. 22–24, 1990.
[6] J. S. Nielsen, W. J. R. Hoefer, "Effect of Dispersion in the 3-D Condensed Node TLM Mesh," in *1992 IEEE Int. Microwave Symp. Dig.*, Albuquerque, NM, June 1–5, 1992.
[7] J. S. Nielsen, "Spurious modes of the TLM-condensed node formulation," *IEEE Microwave Guided Wave Lett*, vol. 1, no. 8, pp. 201–203, Aug. 1991.

Comments on "Josephson Effect Gain and Noise in SIS Mixers"

Daniel G. Jablonski

In their recent article [1], Wengler *et al.* report their observations and conclusions concerning the effects of Josephson currents on the microwave performance of superconductor-insulator-superconductor tunnel junction devices. I would like to caution that the authors seem poised to rediscover, the hard way, many lessons learned by researchers during the 1970s concerning the use of Josephson devices.

First, Wengler *et al.* do not cite any references published prior to 1982. As a result, they make no mention of a considerable body of published work relevant to their current research. In particular, there is no indication that the authors have reviewed early work in the development of Josephson effect mixers [2] and parametric amplifiers [3] built using point-contact and constriction microbridge devices. Point contacts and microbridges were popular because of the difficulties at the time associated with making reliable tunnel junctions, now known as S-I-S devices. Unlike S-I-S devices, point contacts and microbridges have negligible shunt capacitance and do not generally exhibit the quasiparticle, or photon-assisted tunneling steps exploited by S-I-S devices. Users of point contacts and microbridges instead relied on microwave modulation of the Josephson currents within the devices. These currents give rise to the Shapiro steps discussed by Wengler and his coauthors.

For the most part, it was eventually found that microwave applications of Josephson tunneling in point contacts, microbridges, and tunnel junctions were extremely noisy, at least by cryogenic standards. Furthermore, the application of standard microwave theory led to some surprises, particularly with regard to the problem of defining the noise temperature of a Josephson parametric amplifier [4]. It turns out that the gain of such an amplifier depends on the noise spectrum of the input signal. This makes traditional measurements of noise temperature inappropriate. Even though Wengler *et al.* are not observing this mode of operation, it would be wise for them to review the relevant literature, particularly with regard to a problem known as "noise rise." Related to this is the work of Kautz, his colleagues, and others on chaos in Josephson junctions [5].

With respect to their work on S-I-S devices, the authors make no mention of the work of Henneberger and myself on the effects of Josephson currents on the performance of S-I-S devices [6]. If nothing else, this work will make one aware of the many potential difficulties that arise when Josephson steps and quasiparticle steps interact in high frequency, low capacitance devices.

Finally, it should be emphasized that suppressing the Josephson currents is not the same as eliminating the Josephson currents. Even when external Josephson currents are suppressed with a magnetic field, circulating Josephson currents still flow within the S-I-S device. The results of Wengler *et al.* suggest that these circulating currents may significantly degrade the measured signal to noise performance.

Manuscript received June 8, 1992.
The author is with Johns Hopkins University/Applied Physics Laboratory, Johns Hopkins Road, Laurel, MD 20723.
IEEE Log Number 9204037.

References

[1] M. J. Wengler, N. B. Dubash, G. Pance, and R. E. Miller, "Josephson effect gain and noise in SIS mixers," *IEEE Trans. Microwave Theory Tech*, vol. 40, no. 5, pp. 820–826, May 1992.
[2] J. H. Claasen, Y. Taur, and P. L. Richards, "Noise in Josephson point contacts with and without RF bias," *Appl. Phys. Lett.*, vol. 25, pp. 759–761, Dec. 15, 1974.
[3] M. J. Feldman, P. T. Parrish, and R. Y. Chiao, "Parametric amplification by unbiased Josephson junctions," *J. Appl. Phys.*, vol. 46, pp. 4031–4042, Sept. 1975.
[4] S. Wahlsten, S. Rudner, and T. Claeson, "Arrays of Josephson tunnel junctions as parametric amplifiers," *J. Appl. Phys.*, vol. 49, pp. 4248–4263, July 1978.
[5] R. L. Kautz and R. Monaco, "Survey of chaos in the RF-biased Josephson junction," *J. Appl. Phys.*, vol. 57, pp. 875–889, Feb. 1985.
[6] D. G. Jablonski and M. W. Henneberger, "Influence of Josephson currents on superconductor-insulator-superconductor mixer performance," *J. Appl. Phys.*, vol. 58, pp. 3814–3821, Nov. 15, 1985.

Reply to Comments on "Josephson Effect Gain and Noise in SIS Mixers"

Michael J. Wengler

In the above paper[1] we should have placed our work in the context of Josephson mixer work done before 1982. This omission leads to Jablonski's caution. I am pleased to reassure that we are in no danger of rediscovering anything. The earlier work all used point contact junctions with low capacitance and with nonhysteretic current-voltage (IV) curves which fit the resistively shunted Josephson junction (RSJ) circuit model [1], [2]. Our work uses planar SIS diodes with higher capacitance and with completely hysteretic IV's which are not even similar to the RSJ model predictions.

Manuscript received June 19, 1992.
M. J. Wengler is with the Electrical Engineering Department, University of Rochester, Rochester, NY 14627.
IEEE Log Number 9204038.

[1] M. J. Wengler, N. B. Dobash, G. Pance, and R. E. Miller, *IEEE Trans. Microwave Theory Tech.*, vol. 40, no. 5, pp. 820–826, May 1992.